Review Problems for Math 101

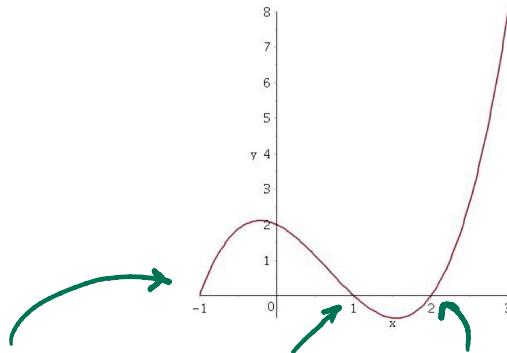1. Let $f(x) = \frac{4}{x+1}$. Evaluate $f(2)$ and $f(a^2+3)$.

$$f(2) = \frac{4}{2+1} = \frac{4}{3} \qquad f(a^3+3) = \frac{4}{(a^2+3)+1} = \frac{4}{a^2+4}$$

2. Below is the graph of the function $f(x) = x^3 - 2x^2 - x + 2$.

(a) Use the graph of $f(x)$ to find the x value(s) where $f(x) = 0$.

$x = -1, 1, 2$

(b) When $1 < x < 2$, is $f(x) > 0$ or $f(x) < 0$?

The graph is below the x-axis so $f(x) < 0$.

3. Is the point $(0,1)$ on the graph of the function $h(x) = \frac{x^2-1}{x^2+1}$?

No $\quad h(0) = \frac{0-1}{0+1} = -1 \neq 1$.

4. Find the points of intersection of the curves $y = x^2 - 4x + 2$ and $y = x - 4$.

$$x^2 - 4x + 2 = y = x - 4$$
$$-x+4 \quad x^2 - 4x + 2 = x - 4 \quad -x+4$$
$$x^2 - 5x + 6 = 0$$
$$(x-3)(x-2) = 0$$
$$x = 2, 3$$

Plug in these x values to get

$y = 2 - 4$; $y = 3 - 4$
$y = -2$; $y = -1$

$(2,-2)$ and $(3,-1)$

5. Simplify the following

(a) $81^{\frac{3}{4}} = \sqrt[4]{81^3} = \left(\sqrt[4]{81}\right)^3 = (3)^3 = \boxed{27}$

(b) $3^{-2} = \frac{1}{3^2} = \boxed{\frac{1}{9}}$

(c) $\frac{f(x+h)-f(x)}{h}$ where $f(x) = x^2 + 2x$.

$= \frac{(x+h)^2 + 2(x+h) - (x^2 + 2x)}{h} = \frac{x^2 + 2xh + h^2 + 2x + 2h - x^2 - 2x}{h}$

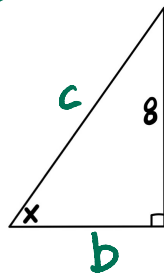$= \frac{2xh + h^2 + 2h}{h} = \frac{\cancel{h}(2x + h + 2)}{\cancel{h}} = \boxed{2x + h + 2}$

6. Find the equation of the line that passes through the point $(3, 2)$ and has a slope of 3.

$y - y_0 = m(x - x_0)$

$\boxed{y - 2 = 3(x - 3)}$ or $\boxed{y = 3x - 7}$

7. If $\tan(x) = \frac{4}{3}$, find the lengths of the other two sides of the given triangle.

$\frac{4}{3} = \frac{8}{6}$ so $\boxed{b = 6}$

Then
$8^2 + 6^2 = c^2$
$64 + 36 = c^2$
$c^2 = 100$ or $\boxed{c = 10}$

8. Find solution(s) to the equation $3x^2 - 2x - 5 = 0$.

$\frac{2 \pm \sqrt{(-2)^2 - 4(3)(-5)}}{2 \cdot 3} = \frac{2 \pm \sqrt{64}}{6} = \frac{2 \pm 8}{6}$ $\quad \frac{10}{6}$ and $\frac{-6}{6}$

$\boxed{\frac{5}{3} \text{ and } -1}$

9. Simplify the following expression by writing it as one fraction:

$\frac{1}{x} - \frac{x+2}{x-1}.$

$\frac{(x-1)}{x(x-1)} - \frac{(x+2)x}{x(x-1)}$

$= \frac{(x-1) - x(x+2)}{x(x-1)} = \frac{x - 1 - x^2 - 2x}{x(x-1)} = \boxed{\frac{-x^2 - x - 1}{x(x-1)}}$