MATH 311 Advanced Linear Algebra

Homework 5

Basic Information

This assignment is due in the correct folder in Google Drive by **4 PM on Friday, February 28**. Any part of the assignment you LaTeX can be turned in by 10 PM without penalty.

Make sure you understand MHC honor code and have carefully read and understood the additional information on the class syllabus and the grading rubric. I am happy to discuss any questions or concerns you have!

You are always welcome to ask me for small hints or suggestions on problems.

Problems

This problem set focuses on the two matrices in P.4.5, so **when I refer to matrix A and B below, I am referring to the matrices in that problem**.

1. Consider the matrices $P_1 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$ and $P_2 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.
   (a) Explain in words exactly what multiplying by each of these matrices does to the matrix A when multiplied on the left (so consider $P_i A$).
   (b) Give examples of two other matrices which do something similar to A also when multiplied on the left.
   (c) What happens when you multiply A by these matrices on the right?

2. For this problem, we will do the LU decomposition of A, but using row reduction as we discussed in class.
   (a) Do the process of row reduction to A **without swapping any rows**. You should not do *reduced echelon form,* and the answer you get should be an upper triangular matrix we'll call U. In preparation for part (b), keep careful track of each step, particularly what you did to get each of the final rows in U
   (b) Next create the lower triangular matrix that represents each step of the row reduction (call it L). For example if you multiplied row 1 by 3 and *subtracted* it from row 2 to get the final row 2 in U, then the second row of L should be [3 1 0].
   (c) Check your final answer is correct by computing the product LU (on computer is fine) and then go back and fix your computations from (a) or (b) as needed if something is wrong. (You don't need to write this part up, just make sure your answers to (a) and (b) are correct before moving on.)

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3. Compute the LU factorization of B either using the way the book describes it, or using the row reduction steps outlined in problem 2 and discussed in class. For the writeup of this problem, you can just show your primary steps algebraically, you do not need to write up an explanation. Check your answer is correct too!

4. LU decomposition is already available as a command in many computer programs. In Python, you can find the algorithm in the scypi.linalg package. I have written up some information on the commands to do this in the Python Guide in Moodle.
   (a) Compute the LU decomposition of A and B using the command mentioned above.
   (b) Does it give you the same answer for L and U as your A and B above? (It probably will not for one of them!)
   (c) For the example where python gives you a different answer than you got in the problems above, look at the output of the first matrix from the lu command (again, see the Python Guide on Moodle for more information) and explain what it is doing to the matrix you are trying to find the LU decomposition of (Hint: Problem 1 is probably related!)

5. Finally, problem P.4.5 asks you to compute the determinant of A and B. What are those determinants and briefly explain why having the LU decomposition of A and B makes them so easy to compute.