MATH 311 Advanced Linear Algebra

Homework 7

Basic Information

This assignment is due in the correct folder in Google Drive by **4 PM on Friday, March 14**. Any part of the assignment you LaTeX can be turned in by 10 PM without penalty. (*For this particular homework, I will not download it before 9 AM on Monday, March 17. So as long as your HW is in the folder by that time it won't be late.*)

Make sure you understand MHC honor code and have carefully read and understood the additional information on the class syllabus and the grading rubric. I am happy to discuss any questions or concerns you have!

You are always welcome to ask me for small hints or suggestions on problems.

Problems

1. P.6.4 The question which is the first sentence of this problem is really the question you will answer by doing (a) and (b), so read the whole problem first and then solve (a) and (b).

2. P.6.22 This problem will be worth 6 points. For each part, briefly show the *core* computations, but make sure you also explain along the way *what* your process was for finding the answer and *why* you used that process. I do not need to see every computational step, but I do want to see sentences (within paragraphs) that explain how and why you solved the problem the way you did with the core computations you choose to show mixed in with the words.

3. Let $\mathscr{V} = M_n(\mathbb{F})$, the vector space of $n \times n$ matrices with entries in $\mathbb{F}$. Consider the linear functional $\mathrm{tr} : \mathscr{V} \to \mathbb{F}$ which sends a matrix A to its trace.
   (a) Explain why we know this map is a linear transformation.
   (b) Construct the Riesz vector for this linear functional using the construction in Theorem 6.4.4. (The inner product for this vector space is defined in Section 5.3.)

4. P.6.9

5. P.7.9

6. (This problem is basically P.7.17.)
   (a) What are all possible entries of a real diagonal $n \times n$ unitary matrix?
   (b) How many such matrices are there? (You may want to think back to your Discrete Math days!)
   (c) What can you say about a *complex* diagonal $n \times n$ unitary matrix?