Math 324 Spring 2017
Homework 6
Due: March 8

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1. Let $F \subseteq K \subseteq L$ be fields. Prove that $[L : F] = [L : K][K : F]$.

2. (a) Prove that if $p > 2$ is a prime and $\omega_p = e^{\frac{2\pi i}{p}}$ then $K = \mathbb{Q}(\omega_p + \omega_p^{-1})$ is an extension of $\mathbb{Q}$ of degree $\frac{p-1}{2}$.

   (b) Prove that K is a subfield of $\mathbb{R}$. This field is called the *maximal real subfield* of $\mathbb{Q}(\omega_p)$.

3. Find the conjugates of $\cos(2\pi/5)$. (It might help to read up on the cosine function in complex analysis.)

4. Prove that $\mathbb{Q}(\sqrt{2}, i\sqrt{2}) = \mathbb{Q}(\sqrt{2} + i\sqrt{2})$.

5. Let $\alpha = a + bi \in \mathbb{C}$ be an algebraic number (so a, $b \in \mathbb{R}$).

   (a) Must a and b be algebraic numbers?

   (b) If α is additionally an algebraic integer must a and b be algebraic integers?

6. Suppose $m \in \mathbb{Z}$ is squarefree.

   (a) What is the field of fractions of $\mathbb{Z} + \mathbb{Z}\sqrt{m}$?

   (b) If also $m \equiv 1 \bmod 4$, prove that $\mathbb{Z} + \mathbb{Z}\sqrt{m}$ is not integrally closed (see definition 4.2.2).

7. *Magma.* It turns out that Magma can easily compute minimal polynomials. If you create the field $K(a)$ (like on last week's homework), it can determine the minimial polynomial of any $b \in K(a)$ using the command `MinimalPolynomial(b)`.

   (a) Write a short program to find the minimal polynomial over $\mathbb{Q}$ of the elements $\omega_p + \omega_p^{-1}$ for any prime p. (I do not need to see a log for all your test cases of this program.)

   (b) Run your program on the first 20 primes greater than 3. Be sure to log all your output from this.